TETHER WAVE EMISSION

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Wave emission is a fundamental effect of electrodynamic tethers [Banks et al., 1981]. The types and frequency ranges of waves emitted, and their impedance, are questions of fundamental importance for both power generation and signal propagation. Here we report briefly on recent results for tethers carrying steady currents [Sanmartín and Martínez-Sánchez, 1994].

Tether wave emission has been studied for more than ten years through a direct use of the linear, Fourier-transformed wave equation,

$$\frac{-\overline{k}_\wedge(\overline{k}_\wedge\overline{E})}{k^2} - \frac{\overline{\overline{\epsilon}}_c \cdot \overline{E}}{n^2} = \frac{4\pi i \overline{j}_s}{\omega n^2}. \tag{1}$$

Here n and $\overline{\overline{\epsilon}}_c$ are the refractive index and dielectric tensor of a cold magnetoplasma,

$$n = \frac{ck}{\omega} \quad , \qquad \overline{\overline{\epsilon}}_c(\omega) \equiv \begin{pmatrix} \epsilon_1 & i\epsilon_2 & 0 \\ -i\epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix}.$$

Different models for the (source) tether current density $\overline{j}_s$ have been used in the literature. Note, however, that Eq. (1) does not take into account nonlinear effects present near the tether.

The well known (Aström) dispersion relation for Eq. (1) may be written as

$$\begin{aligned} &\left(\epsilon_1 \sin^2\theta + \epsilon_3 \cos^2\theta - \frac{\epsilon_3\epsilon_1}{n^2}\right)\left(1 - \frac{\epsilon_1}{n^2}\right) + \\ &\qquad + \left(\sin^2\theta - \frac{\epsilon_3}{n^2}\right)\frac{\epsilon_2^2}{n^2} = 0, \end{aligned} \tag{2}$$

where θ is the angle between $\overline{B}$ (along axis z) and $\overline{k}$. Aström's dispersion relation is a 5^{th} degree equation for $\omega^2(k, \theta)$. The 5 wave branches are usually called Fast Extraordinary (FE), Ordinary (O), Slow Extraordinary (SE), Fast Magnetosonic (FM) and Alfven (A).

To determine which waves are radiated we first recall conditions appropriate to tethers. The Doppler condition reads

$$\omega = k_x V$$

where V is orbital velocity, and we took, as usual, axes x and y along orbit and tether respectively. In addition, if $V_A \equiv c\Omega_i/\omega_{p_i}$ (ω_{p_i} and Ω_i being ion plasma frequency and gyrofrequency respectively) is the Alfven speed, we have

$$V \ll V_A \ll c \quad ,$$

leading to

$$n = \frac{c}{V}\frac{k}{k_x} \gg \frac{c}{V_A} \gg 1 \tag{3}$$

($V \simeq 7km/s$, $V_A \sim 300km/s$). We also have

$$V_A^2 \ll V^2 m_i/m_e \qquad (m_i/m_e \simeq 30.000). \tag{4}$$

We then note that i) FE and O waves in Eq. (2) have $n < 1$; it follows from (3) that there can be no emission on those branches. ii) SE and FM waves in Eq. (2) have asymptotes $k \to \infty$ ($n \to \infty$), with ϵ_1/n^2, ϵ_2/n^2 and $\epsilon_3/n^2 \to 0$, given by the limit equation from (2)

$$\epsilon_1(\omega)\sin^2\theta + \epsilon_3(\omega)\cos^2\theta = 0, \tag{5}$$

yielding roots $\omega_{SE}(\infty,\theta)$ and $\omega_{FM}(\infty,\theta)$; it follows from (3) that SE and FM emission occur near the respective asymptote. For the FM branch, having $\omega \simeq \omega_{FM}(\infty,\omega)$ actually requires condition $n \gg c/V_A$ (or $V \ll V_A$) rather than just $n \gg 1$. Note that there can be no emission of whistlers, which correspond to another part of the FM branch and require condition $V \gg V_A$, as in recent experiments by Stenzel and Urrutia [1990] with $V \simeq 2\times 10^7 cm/s \gg V_A \simeq 4\times 10^5 cm/s$, which are thus not appropriate for Earth tethers.

Finally, iii) in the Alfven branch, condition $n \to \infty$ may result from the limit $\omega \to 0$ instead of $k \to \infty$. For $\omega \to 0$ we have $|\epsilon_3| \to \infty$, and $|\epsilon_3|/n^2$ may take any value. For $|\epsilon_3|/n^2$ small, Alfven waves would again be given by (5), which is a cubic equation: $\omega \simeq \omega_A(\infty,\theta)$. Actually, however, condition (4) makes $|\epsilon_3|/n^2$ large for tethers,

$$\frac{|\epsilon_3|}{n^2} \simeq \frac{\Omega_i^2}{\omega^2}\frac{k_x^2}{k^2}\frac{V^2 m_i}{V_A^2 m_e} \gg 1,$$

except for a narrow wave vector range carrying negligible energy. A emission thus occurs at the special limit $|\epsilon_1|$, $|\epsilon_2| \ll n^2 \ll |\epsilon_3|$, with a frequency given by the limit dispersion relation

$$\epsilon_3\cos^2\theta - \frac{\epsilon_1\epsilon_3}{n_2} = 0\ , \qquad (\cos^2\theta \ll 1).$$

With the above results, Eq. (1) can be solved for the potential ϕ of the longitudinal part of field $\overline{E}$

$$\frac{\overline{k}\,\overline{k}}{k^2}\cdot\overline{E} \equiv \overline{E}_l = -i\overline{k}\phi.$$

Using $\overline{k}\cdot(\overline{E}-\overline{E}_l) = 0$, one finds $\phi \propto \overline{k}\cdot\overline{j}_s$ for all three branches SE, FM and A, with $\overline{E} = \overline{E}_l + 0(n^{-2})$ for SE and FM, and $\overline{E} = \overline{E}_l - \overline{E}_l\cdot\overline{B}/B + 0(n^{-2})$ for A. Details are given elsewhere [Sanmartín and Martínez-Sánchez, 1994].

The impedanze Z is then immediately obtained from the expresion

$$\text{Power radiated} = -\int \overline{j}_s\cdot\overline{E}d\overline{r} = -\int \phi\nabla\cdot\overline{j}_s d\overline{r}\ ,$$

using Fourier transforms and the relation between ϕ and $\overline{k}\cdot\overline{j}_s$:

$$Z = \frac{\text{Power}}{\text{I}^2} = \int \frac{2id\overline{k}}{Vk_x}|g(\overline{k})|^2\left[(k_x^2+k_y^2)\varepsilon_1(k_xV) - k_z^2\left\{\begin{matrix} -\epsilon_3(k_xV) \\ \\ \dfrac{c^2}{V^2}\dfrac{k_x^2+k_y^2}{k_x^2}\end{matrix}\right\}_A^{SE,FM}\right]^{-1}.$$

Here,

$$g(\overline{k}) \equiv \frac{-i}{2\pi}\int d\overline{r}\frac{\nabla\cdot\overline{j}_s(\overline{r})}{I}exp(-i\overline{k}\cdot\overline{r})\ ,$$

I being the current through the tether.

Usual impedance studies consider tether end contactors with source divergence $\nabla \cdot \bar{j}_s$ having zero characteristic length L_y along the tether. One may then write

$$g(\bar{k}) = \frac{1}{\pi} \sin\left(\frac{k_y L}{2}\right) \times g_{per}(\bar{k}_{per})$$

where L is the tether length, $\bar{k}_{per} \equiv (k_x, k_z)$, and g_{per} is the average of $exp(-i\bar{k}_{per} \cdot \bar{r})$ on the contactor. Examples are

$$g_{per} = \frac{\sin(k_x L_x/2)}{k_x L_x/2} \quad , \qquad \text{[Estes, 1988]}$$

$$g_{per} = \frac{J_1(k_{per} R)}{k_{per} R/2} \quad , \qquad \text{[Barnet and Olbert, 1986]}$$

$$g_{per} = \frac{\sin(k_x L_x/2)}{k_x L_x/2} \frac{\sin(k_z L_z/2)}{k_z L_z/2} \quad , \qquad \text{[Hastings et al., 1988]}$$

the respective contactors being a segment L_x along the x axis, a circle of radius R, and a rectangular surface of sides L_x and L_z.

For the Alfven branch we have $\bar{k}_{per} \cdot \bar{r} \simeq 0$ and all models yield $g_{per} \simeq 1$ leading to the known result,

$$Z_A = \frac{2V_A}{c^2} [\ln 2 + \gamma - 1 + \ln(\Omega_i L/V)] \quad , \quad \gamma \simeq 0.577 \, . \tag{6}$$

For the SE and FM branches, however, we have $|\bar{k}_{per} \cdot \bar{r}| \gg 1$, and both $|g|^2$ and Z scale differently with size for different contactor models: Z (Estes) $\sim 1/L_x^2$, Z (Barnett & Olbert) $\sim 1/R^3$, Z (Hastings et al.) $\sim 1/L_x^2 L_z^2$. A proper model should have a length $L_y \neq 0$. For spherical contactors we find

$$g = \frac{1}{\pi} \sin\left(\frac{k_y L}{2}\right) \frac{\sin kR}{kR} \, ,$$

recovering Eq. (6) for the A branch, and

$$g \propto 1/R^2 \propto 1/\text{Contactor Area}$$

for both the SE and FM branches.

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